and, consequently, the $\triangle MNO=\frac{1}{8}(\mathbf{N})^2 / \sin A \sin B \sin C \ldots (5)$. The expression for the average area of the $\triangle MNO$, therefore, becomes

$$\mathbf{A}=\frac{1}{8abc \sin A \sin B \sin C}\int_0^a\int_0^b\int_0^c(\mathbf{N})^2 dxdydz$$

$$=\tfrac{1}{24}\left(\frac{a^2 \sin A}{\sin B \sin C}+\frac{b^2 \sin B}{\sin C \sin A}+\frac{c^2 \sin C}{\sin A \sin B}\right)=\frac{a^4+b^4+c^4}{48\triangle}.$$

[Note.—Problems twenty-three and twenty-four are identical. This fact was not observed until after they were both printed. Ed.]

PROBLEMS.

31. **Proposed by B. F. FINKEL, A. M., Professor of Mathematics and Physics in Drury College, Springfield, Missouri.**

Find the average length of a line drawn at random across the opposite sides of a rectangle whose length is l and breadth b.

32. **Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.**

Find the average area of the random sector whose vertex is a random point in a given circle.

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

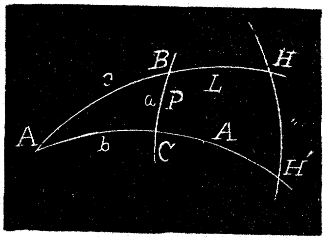
SOLUTIONS OF PROBLEMS.

17. **Proposed by SAMUEL HART WRIGHT, M. D., M. A., Ph. D. Penn Yan, New York.**

A bright star passed my meridian at 7 P. M. The Chronometer soon after ran down and stopped, but I set it again when the same star had a true altitude of $30^\circ=\alpha$. What time was it then, my latitude being $42^\circ\ 30'$ N.$=\lambda$, and the star's declination 60° N.?$=\delta$

II. **Solution by the PROPOSER.**

Let B be the north pole, A the zenith, C the star, HH' the horizon, AH and AH' each $=90^\circ$, AH' being a meridian, AH a verticle circle, BH' the altitude of the pole $=$ the latitude $=L$, $AB=$co-latitude $=c$, $BC=a=$ polar distance of the star$=P$, $AC=b=$the zenith distance of star, $CH=A=$altitude of the star, and the angle $ABC=$the hour-angle of the star$=T$ in siderial time. Put $s=\frac{1}{2}(a+b+c)$, and $s-a=a'$, $s-b=b'$, and $s-c=c'$. Then by *Sph. Trig.*



$\sin \frac{1}{2}T=\sqrt{\left(\frac{\sin c' \sin a'}{\sin c \sin a}\right)}$, and

$\frac{1}{2}T=51^\circ 40' 18''.5\ldots(1)$ or $\cos\frac{1}{2}T=\sqrt{[\sin s \sin(s-b) \operatorname{cosec} c \operatorname{cosec} a]}$, and $\frac{1}{2}T=51^\circ 40' 18''.5\ldots(2)$, or in terms of A, L, and P; put $s'=\frac{1}{2}(A+L+P)$, then $s=\frac{1}{2}(180^\circ-A-L+P)=90^\circ-s'+P=90^\circ-A-L+s'$, and $s-P=90^\circ-s'$, and $s-(90^\circ-L)=s'-A$. Whence, $\sin\frac{1}{2}T=\sqrt{[\sec L \operatorname{cosec} P \cos s' \sin(s'-A)]}$, and $\frac{1}{2}T=51^\circ 40' 18''.5\ldots(3)$.

$\therefore$ $T=103^\circ 20' 37''=6$ hr. 53 min. 22.467 sec. of siderial time=6 hr. 52 min. 14.75 sec. mean solar time. To this add 7 hrs., the time the star was on the meridian, and we get 1 hr. 52 min. 14.75 sec. of the morning of the next day, for the time when the chronometer was set.

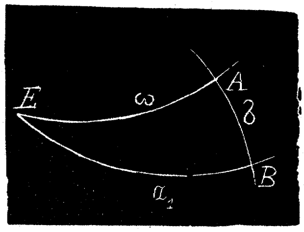
In Bowditch's *Practical Navigator*, pp. 209-210, the rules for finding T are translations of eqs. (2) and (3), but no reasons for the rules are given, and no formulas from which they are derived. The above formulas, (1), (2), and (3) are as applicable for obtaining correct time on land as at sea. [This solution is important as showing how the Rule in Bowditch's *Navigator* is obtained,—which some very good mathematicians have failed to comprehend.—EDITOR.]

20. Proposed by SAMUEL HART WRIGHT, M D., M. A., Ph. D., Penn Yan, New York.

When does the Dog-Star and the Sun rise together in latitude $42^\circ 30'$ N. $=\lambda$, given the R. A. of Sirius=6 hrs. 40 min. 30 sec., and its Dec. $=16^\circ 33' 56''$ S.?

II. Solution by G. B. M ZERR, A. M, Ph. D, Professor of Mathematics and Science in Inter State College, Texarkana, Texas.

Let $\lambda=$ latitude of observer, $a=$R. A., $\delta=$declination, $t=$hour angle of Sirius. $a_1=$R. A., $\delta_1=$declination, $t_1=$hour angle of sun, $\varepsilon=$obliquity of the ecliptic, $\omega=$ distance from vernal equinox to the sun's position, $\tau=$ time of sun-rise before six o'clock. Then we get $\cos t=-\tan\lambda\tan\delta\ldots(1)$. $\cos t=-\tan\lambda\tan\delta_1\ldots(2)$. $\sin a_1=\tan\delta_1 \cot\varepsilon\ldots(3)$. $a_1-t_1=a-t=\theta$, or $a_1=\theta+t_1 \ldots(4)$. $\sin a_1=\sin(\theta+t_1)\ldots(5)$. $\cos\varepsilon=\cot\omega \tan a_4\ldots(6)$. $\sin\tau=\tan\lambda\tan\delta_1\ldots(7)$. From (3) and (5), $\sin(\theta+t_1)=\tan\delta_1\cot\varepsilon\ldots(8)$. From (2) and (8),



$$\tan\delta_1=\frac{\sin(\theta+t_1)}{\cot\varepsilon}=-\frac{\cos t_1}{\tan\lambda}\ldots(9).$$

From (9), $$\tan t_1=-\frac{\cot\varepsilon+\sin\theta\tan\lambda}{\cos\theta\tan\lambda}\ldots(10).$$

But $\lambda=42^\circ 30'$, $a=6$ hr. 40 min. 30 sec., $\delta=16^\circ 33' 56''$ S., $\varepsilon=23^\circ 27' 13''$.

From (1), $t=74^\circ 10' 57''.81=4$ h. 56 m. 43.85 sec.

$a-t=\theta=1$ h. 43 m. 46.15 sec. $=25^\circ 56' 32''.25=$siderial time when Sirius rises.

From (10), $t_1=107^\circ 3' 20''=7$h. 8 m. 13.33 sec.

From (4), $a_1=t_1+\theta=8$ h. 51 m. 59.48 sec. $=132^\circ 59' 52''.25$.

From (3), $\delta_1=17^\circ 36' 16''.85$. From (6), $\omega=130^\circ 32' 38''.5$.